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PELREF
A numerical code for computing the ablated state of a refuelling pellet

C.T. Chang

Abstract. Assuming a constant specific heat ratio and using the neutral shileding model, this report presents a numerical code for calculating the ablation rate and the state of the ablatant of a refuelling pellet. Results are given for plasma conditions corresponding to present and to future toridal devices.

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CONTENTS

1. INTRODUCTION

A numerical code for computing the ablation rate of a refuelling pellet, clearly should exist among a few laboratories. For various purposes, as, for example, the development of a diagnostic method in evaluating the pellet local ablation rate, the calculating of the fuel transport rate, and the possible modification and extension of the existing neutral shielding model[(1)], etc., it should be desirable to have such a computational code at our own disposal.

This report summarizes the result of such an effort. Section 2 first gives a brief description of the neutral shielding model relevant to the numerical analysis. This is followed by a derivation of the asymptotic solution of the ablated flow. Section 3 gives a brief description of the code and the general computational results obtained. A detailed description of a typical program is included in the appendix A. Examples of computational results are given for a present experimental and a future large toridal device.

2. THE MODEL

2.1. Governing equations

According to the neutral-shielding model[(1),(2)] of a refuelling pellet, the kinetic energy of the ablated flow is described by the equation

$$\frac{dw}{dr} = \frac{2w}{(\theta-w)} \left[A_* r^2 \frac{dq}{dr} - 2\,\frac{\theta}{r}\right] \tag{1}$$

where $w = v^2$ and θ are the kinetic energy density and temperature normalized with respect to their values at the sonic radius,

r_*. Similarly, r is the normalized radial coordinate; thus, at the sonic radius, r = 1. The pellet surface is denoted by $\hat{r}=r_p/r_*$.

The dimensionless input rate of the electron energy is given by

$$\frac{dq}{dr} = \frac{\rho_* \Lambda_* r_*}{m_{H_2}} \rho \Lambda'(E) q \quad , \tag{2}$$

where $\Lambda'(E) = \Lambda(E)/\Lambda_*$ is the normalized "total energy loss cross section". The total energy loss cross section $\Lambda(E)$ has two terms: The inelastic collision term is represented by the stopping cross section $2L(E)/E$; the elastic collision is represented by the scattering cross section, $\sigma(E)$. Thus

$$\Lambda(E) = 2\,\frac{L(E)}{E} + \sigma(E) \quad , \tag{3}$$

where

$$L(E)[\text{eV-cm}^2] = \{2.35\times10^{14} + 4\times10^{11}E + 2\times10^{17}E^{-2}\}^{-1} \;; \tag{4a}$$

$$\begin{aligned}\sigma(E)[\text{cm}^2] &= 8.8\times10^{13}E^{-1.71} - 1.62\times10^{-12}E^{-1.932} \\ &\qquad\qquad \text{for } E \geq 100 \text{ eV} \;, \\ &= 1.13\times10^{-14}E^{-1} \\ &\qquad\qquad \text{for } E < 100 \text{ eV} \;.\end{aligned} \tag{4b}$$

The parameter A_*, which appeared in Eq. (1), is a constant depending on physical parameters at the sonic radius and the mass ablation rate G. Thus

$$A_* = \frac{4\pi f_B Q\left(\frac{\gamma-1}{\gamma}\right) q_* r_*^2}{kT_* G}\, m_{H_2} \quad , \tag{5}$$

where f_B and Q are reduction factors of the incident electron energy flux due to the magnetic field effect and the inelastic collision losses occurring in the ablated cloud. For simplicity, in the original model[1] f_B and Q were taken as constants; $f_B \simeq 0.5$, $Q \simeq 0.6$.

If we seek a solution of a continuously accelerated flow of the ablatant, i.e. $\frac{dw}{dr} > 0$ everywhere and the requirement at the far downstream $\left(\frac{dq}{dr}\right)_{r\to\infty} \to 0$, we observe that from Eq. (1) the flow downstream at a great distance must be supersonic, i.e. $w > \theta$. On the other hand, since we expect the temperature of the ablatant to be low near the pellet surface, the flow must be subsonic there, i.e. $w < \theta$.

At the sonic radius, since $\theta = w$ we must impose the condition to make the terms in the square bracket of Eq. (1) vanish. As a result, we obtain

$$2\pi f_B Q\left(\frac{\gamma-1}{\gamma}\right)\frac{q_* \rho_* r_*^3 \Lambda_*}{kT_* G} - 1 = 0 \ . \qquad (6)$$

Using Eq. (6) and the mass conservation equations

$$\rho v r^2 = \rho\sqrt{w} r^2 = 1 \ .$$

Eq. (1) can be reduced to

$$\frac{dw}{dr} = \frac{4w\theta}{(\theta-w)r}\left[\frac{\Lambda' q r}{\theta\sqrt{w}} - 1\right] \ . \qquad (1a)$$

For the purpose of numerical computation, it would be more appropriate to replace w by $v = \sqrt{w}$, the governing equations of the ablated flow can then be written as follows:

$$\frac{dv}{dr} = \frac{2v\theta}{(\theta-v^2)}\left[\frac{\Lambda' q}{\theta v} - \frac{1}{r}\right] \ , \qquad (7)$$

$$\frac{d\theta}{dr} = 2\frac{\Lambda' q}{v} - (\gamma - 1)v\frac{dv}{dr} \ , \qquad (8)$$

$$\frac{dq}{dr} = \lambda_* \frac{\Lambda' q}{r^2 v} \ , \qquad (9)$$

$$\frac{dE'}{dr} = 2\lambda_*\left[\frac{L(E)}{E_* \Lambda_*}\right] \frac{1}{r^2 v} \ , \qquad (10)$$

$$\rho v r^2 = 1 \ , \qquad (11)$$

$$p = \rho\theta \ . \qquad (12)$$

The Mach number of the flow is given by

$$M = (v^2/\theta)^{\frac{1}{2}} \ . \tag{13}$$

All parameters except E in the above system of equations are normalized with respect to their values at the sonic radius. The incident electron energy at the normalized sonic radius $r = 1$ is denoted by E_*; thus, $E' = E/E_*$ is the normalized electron energy. Notice that $L(E)/E_*$ has the dimension of a cross section, and $L(E)/E_*\Lambda_*$ is dimensionless.

The above system of equations contains two parameters, λ_* and Λ_*. They are defined as

$$\lambda_* = \rho_*\Lambda_* r_*/m_{H_2} \ , \tag{14}$$

$$\Lambda_* = \Lambda(E_*) \ . \tag{15}$$

Both Λ_* and λ_* depend on E_*, the incident electron energy at the sonic radius. As indicated by Eq. (3) and Eqs. (4a) and (4b), once E_* is chosen, $\Lambda_* \equiv \Lambda(E_*)$ is fixed; however λ_*, to be shown subsequently, varies within a limited range.

2.2. Eigenvalue of the problem

At the sonic radius of $r = 1$, all flow parameters and Λ' become unity while $\frac{dv}{dr}$ becomes indeterminate. For a given value of E_*, this derivative

$$Z_s = \left(\frac{dv}{dr}\right)_{r=1} \tag{16}$$

as will be shown, depends on the parameter λ_*, which therefore can be viewed as an eigenvalue of the problem.

From Eqs. (8-10), we have at $r = 1$

$$\frac{d\theta}{dr} = 2 - (r-1)Z_s \ , \tag{17}$$

$$\frac{dq}{dr} = \lambda_* , \tag{18}$$

$$\frac{dE}{dr} = 2\,\frac{\Lambda_* L(E_*)}{E_* \Lambda_*} , \tag{19}$$

whereas Z_s can be determined from Eq. (7) by applying the L'Hôpital's rule. This gives a quadratic equation in Z_s; its explicit solution is given by

$$Z_s = \left(\frac{3-\gamma}{1+\gamma}\right)\left\{1 \pm \sqrt{1 - 2\frac{(1+\gamma)}{(3-\gamma)^2}(\lambda_* + \psi_* - 1)}\right\}, \tag{20}$$

where

$$\psi_* = \left(\frac{d\Lambda'}{dr}\right)_{r=1} \tag{21}$$

and can be written as

$$\begin{aligned} \psi_* &= \frac{E_*}{\Lambda_*}\left(\frac{d\Lambda}{dE}\right)_{E=E_*} \cdot \left(\frac{dE}{dr}\right)_{r=1} , \\ &= 2\lambda_* \frac{L(E_*)}{\Lambda_*^2}\left(\frac{d\Lambda}{dE}\right)_{E=E_*} . \end{aligned} \tag{22}$$

Introducing

$$N_* = 2\,\frac{L(E_*)}{\Lambda_*^2}\left(\frac{d\Lambda}{dE}\right)_{E=E_*} \tag{23}$$

we may further write $\psi_* = N\lambda_*$ and replace the eigenvalue, λ_*, by

$$K = (1 + N_*)\lambda_* . \tag{24}$$

The derivative $Z_s = \left(\frac{dv}{dr}\right)_{r=1}$ now is related to K by

$$Z_s = \left(\frac{3-\gamma}{1+\gamma}\right)\left\{1 \pm \sqrt{1 + 2\frac{(1+\gamma)}{(3-\gamma)^2}(1 - K)}\right\} . \tag{25}$$

As a first requirement for Z_s to be real, it is necessary that

$$2\frac{(1+\gamma)}{(3-\gamma)^2}(K-1) < 1 . \tag{26}$$

However, if we expect a solution with $\frac{dv}{dr} > 0$ in the entire flow, Z_s must be positive. Furthermore, if we require the uniqueness of the positive root (i.e., the two real roots of z_s are of different sign) we must impose the condition that

$$1 + 2\frac{\gamma+1}{(3-\gamma)^2}(1-K) > 1 \qquad \text{or } K < 1 \ .$$

Stated briefly, after a value of E_* is assigned, according to Eq. (23) we find that N_* is fixed, the proper value of λ_* must be chosen such that K is within the open interval (0,1).

2.3. Boundary conditions

In view of the low sublimation energy of solid deuterium and the expectation of the existence of a self-shielding mechanism, the boundary conditions at the pellet surface $\hat{r} = r_p/r_*$ may be written as

$$\left.\begin{aligned} q(\lambda_*,E_*) &= 0 \qquad (27) \\ \theta(\lambda_*,E_*) &= 0 \qquad (28) \end{aligned}\right\} \text{ at } r = \hat{r} \ .$$

The boundary conditions at the far distance region downstream clearly can be taken as

$$\left.\begin{aligned} q(\lambda_*,E_*) &\to \tilde{q} = q_o/q_* \\ E(\lambda_*,E_*) &\to \tilde{E} = E_o/E_* \end{aligned}\right\} \text{ at } r \geq r_m \ , \qquad (30)$$

or alternatively,

$$\frac{dq}{dr} \to 0, \ \frac{dE}{dr} \to 0 \text{ for } r \geq r_m \ . \qquad (31)$$

By specifying these boundary conditions and defining all the derivatives in terms of the given value E_* and a guessed eigenvalue λ_*, we may then proceed the integration from the sonic surface inward to see whether the inner boundary conditions are satisfied at the pellet surface. For a given E_* the process has to be repeated by guessing the value λ_* until Eqs. (27) and (28) are satisfied. In other words, Eqs. (27) and (28) really are used to locate the pellet surface. Similarly, we may consider the outer boundary conditions as a means of locating the ablated cloud boundary.

2.4. Flow parameters at the sonic radius

Since all the flow parameters thus far were normalized with respect to their values at the sonic radius, to obtain their actual physical values it is necessary to determine their corresponding values at the sonic radius first. Using the facts that the flow is sonic, the mass is conserved, and the state of the ablatant can be taken as an ideal gas, we have the following three algebraic equations:

$$v_* = (\gamma k T_*/m_{H_2})^{\frac{1}{2}} , \tag{32}$$

$$\rho_* v_* r_*^2 = G/4\pi , \tag{33}$$

$$p_* = \rho_* k T_*/m_{H_2} . \tag{34}$$

These equations together with the two auxilliary equations:

$$f_B Q\left(\frac{\gamma-1}{r}\right) q_* \rho_* \Lambda_* r_*^3 = kT_* \frac{G}{2\pi} \tag{6}$$

and

$$\lambda_* m_{H_2} = \rho_* \Lambda_* r_* \tag{14}$$

form a system of five equations for the eight unknowns; G, ρ_*, p_*, T_*, v_*, r_*, q_* and E_* (recalling that both Λ_* and λ_* depend on E_*). If we consider E_*, r_*, and q_* as given, they can be solved explicitly

$$\frac{G}{4\pi} = \frac{\lambda_* m_{H_2}^{2/3} [f_B Q(\gamma-1) q_*]^{1/3} r_*^{4/3}}{(2\Lambda_*^2)^{1/3}} , \tag{35}$$

$$\rho_* = \frac{\lambda_* m_{H_2}}{\Lambda_* r_*} , \tag{36}$$

$$kT_* = \frac{m_{H_2}^{1/3}}{\gamma} \left[\frac{f_B Q(\gamma-1) q_* r_* \Lambda_*}{2}\right]^{2/3} , \tag{37}$$

$$v_* = \left[\frac{f_B Q(\gamma-1) q_* r_* \Lambda_*}{2\ m_{H_2}}\right]^{1/3} , \tag{38}$$

$$\gamma p_* = \left(\frac{m_{H_2}}{\Lambda_* r_*}\right)^{1/3} \left[\frac{f_B Q(\gamma-1) q_*}{2}\right]^{2/3} \lambda_* . \tag{39}$$

Since the factor $f_B Q$ always appears together with q_*, we may absorb it into q_*. We recal that r_* and E_* are related to the pellet radius r_p and the unattenuated electron energy E_o (or the plasma temperature $2kT_o$) by

$$r_* = r_p/\hat{r} , \tag{40}$$

$$E_* = E_o/\tilde{E} = 2\ kT_o/\tilde{E} , \tag{41}$$

and

$$q_* = q_o/\tilde{q} = [n_o(4\pi m_e)^{-\frac{1}{2}}(2kT_o)^{3/2}]/\tilde{q} . \tag{42}$$

These expressions, Eqs. (35-39), in turn can be related to the plasma parameters n_o, kT_o and the pellet radius, r_p. For example, we have explicitly,

$$G = 8.4125\times10^{-17}(\gamma-1)^{1/3} \frac{\lambda_*}{\Lambda_*^{2/3}} \frac{n_o^{1/3}(kT_o)^{1/2}}{\tilde{q}^{1/3}} \left(\frac{r_p}{\hat{r}}\right)^{4/3} , \tag{35a}$$

$$\rho_* = 3.3452\times10^{-24}\left(\frac{\lambda_*}{\Lambda_*}\right) \frac{\hat{r}}{r_p} \tag{36a}$$

and

$$kT_* = \frac{8.3592}{\gamma}\left[(\gamma-1) n_o\left(\frac{r_p}{\hat{r}}\right) \frac{\Lambda_*}{\tilde{q}}\right]^{2/3} (kT_o) . \tag{37a}$$

(All quantities appearing above are in C.G.S. units except for kT_o and kT_* which are in eV).

On account of the dependence of λ_*, Λ_*, $\tilde{q}$ and $\hat{r}$ on E_*, these equations indicate that for a given pellet radius r_p and the ambient plasma condition of n_o and kT_o, the mass ablation rate, G, and the state of the ablatant, ρ_* and kT_*, are still undetermined. In practice, this implies that based on the expected value of $\tilde{E}$, one must first guess a proper value of E_* ($E_* = 2kT_o/\tilde{E}$), and calculate $\hat{r},\tilde{q}$ and $\tilde{E}$ which, in turn, give a set of calculated r_p, q_o, and kT_o. The process is to be iterated until the calculated values of r_p, q_o, and kT_o agree reasonably well with their initial given values.

2.5. The asymptotic solutions

From the computational results, we notice that once the ablated flow passes beyond the sonic radius, both the electron energy flux q and the energy E' approach their corresponding values of the ambient plasma quickly. This indicates that asymptotic solutions of the flow parameters might exist at a sufficiently larger value of r.

To study this possibility, we introduce a new variable

$$\mu = v^2/\theta, \tag{43}$$

or $\mu \equiv M^2$, to replace the dependent variable θ. The system of differential equations, Eqs. (7)-(10), now takes the following form:

$$\frac{dv}{dr} = \frac{2V}{1-\mu}\left[\frac{\Lambda' q\mu}{v^3} - \frac{1}{r}\right], \tag{44}$$

$$\frac{d\mu}{dr} = \left[\frac{(\gamma-1)\mu^2+2\mu}{v}\right]\frac{dv}{dr} - 2\Lambda'\frac{q\mu^2}{v^3}, \tag{45}$$

$$\frac{dq}{dr} = \lambda_*\frac{\Lambda' q}{r^2 v}, \tag{46}$$

$$\frac{dE'}{dr} = 2\lambda_*\left[\frac{L(E)}{E_*\Lambda_*}\right]\frac{1}{r^2 v}. \tag{47}$$

Assuming that λ, q, and μ are slowly varying functions of r, for $r \gg r_o$, we may put

$$1-\mu = A(r), \quad \Lambda' q\mu = B(r) \tag{48}.$$

Eq. (44) may then be written as

$$\frac{dV}{dr} = 2\,\frac{V}{A}\left[\frac{B}{V^3} - \frac{1}{r}\right] \quad . \tag{49}$$

As a first approximation, neglecting $1/r$ and considering A and B as constants, we have

$$\frac{dv}{dr} = 2\,\frac{B}{A}\,\frac{1}{v^2} \qquad \text{or } v^3 \sim r \quad ,$$

we then may put

$$v(r) = y(r)r^{1/3} . \tag{50}$$

After substituting this into Eq. (49), we obtain

$$\frac{dy}{2\,\frac{B}{A}\,\frac{1}{y^2} - 2\left(\frac{1}{A}+\frac{1}{6}\right)y} = \frac{dr}{r} \quad . \tag{51}$$

Introducing $C_1 = 2\frac{B}{A}$, $\qquad C_2 = -2\left(\frac{1}{A} + \frac{1}{6}\right)$

then

$$\frac{C_1}{C_2} = -\,\frac{B}{(1 + \frac{A}{6})}$$

$$= -\,6\,\frac{\Lambda' q\mu}{7-\mu} \quad . \tag{52}$$

Denoting all asymptotic values of the variables concerned by "~", we now impose the condition that for $r >> r_o$

$$\frac{C_1}{C_2} = -\,6\,\frac{\tilde{\Lambda}'\,\tilde{q}\,\tilde{\mu}}{7-\tilde{\mu}} = \text{constant.} \tag{52a}$$

Eq. (51) can be written as

$$\frac{\frac{3y^2dy}{C_1}}{\frac{C_1}{C_2} + y^3} = 3\,C_2\,\frac{dr}{r} \quad . \tag{53}$$

Carrying out the integration from r_o to r, we obtain

$$\frac{C_1/C_2 + y^3}{C_1/C_2 + y_o^3} = \left(\frac{r}{r_o}\right)^{3C_2} .$$

Solving the above equation for y^3, we find

$$y^3 = \left(\frac{r}{r_o}\right)^{3C_2}\left[\left(\frac{C_1}{C_2}\right) + y_o^3\right] - \frac{C_1}{C_2} . \tag{54}$$

If we require that y = constant for $r > r_o$, we have

$$y_o^3 = \frac{v_o^3}{r_o} = -\frac{C_1}{C_2} = 6\frac{\tilde{\Lambda}\,\tilde{q}\,\tilde{\mu}}{7-\tilde{\mu}} , \tag{55}$$

then

$$y = \left(-\frac{C_1}{C_2}\right)^{1/3} = \left(\frac{6\,\tilde{\Lambda}\,\tilde{q}\,\tilde{\mu}}{7-\mu}\right)^{1/3} , \tag{56}$$

or

$$\tilde{v}(r) = \left(\frac{6\tilde{\Lambda}\,\tilde{q}\,\tilde{\mu}}{7-\mu}\right)^{1/3} r^{1/3} . \tag{57}$$

From Eq. (45), when $\mu \to \tilde{\mu}$, or $\frac{d\mu}{dr} = 0$, we have

$$\tilde{v}^2\,\frac{d\tilde{v}}{dr} = \frac{\Lambda' q}{\frac{\gamma-1}{2} + \frac{1}{\tilde{\mu}}} . \tag{58}$$

Substituting Eq. (57) into (58), after rearranging the terms, we obtain finally

$$\tilde{\mu} = 5/\gamma . \tag{59}$$

Using Eq. (57) we eliminate $\tilde{v}$ in (58) and get

$$\tilde{v} = \left(\frac{30\tilde{\Lambda}\tilde{q}}{7\gamma-5}\right)^{1/3} r^{1/3} . \tag{60}$$

Similarly

$$\tilde{\theta} = \frac{\tilde{v}^2}{\tilde{\mu}} = \frac{\gamma}{5}\left(\frac{30\tilde{\Lambda}\tilde{q}}{7\gamma-5}\right)^{1/3} r^{2/3} , \tag{61}$$

$$\tilde{\rho} = \frac{1}{\tilde{v}r^2} = \left(\frac{30\tilde{\Lambda}\tilde{q}}{7\gamma-5}\right)^{-1/3} r^{-7/3} \tag{62}$$

3. COMPUTATIONAL CODE AND GENERAL RESULTS

To faciliate numerical computations, as shown in Table I, a change of notation was made. Notice that the dependent variable v, θ, q, E and their derivatives are denoted by Y1, Y2, Y3, Y4 and Z[1], z[2], z[3], z[4] in the procedure F(X, Y, Z), they are denoted by YY[1,1], YY[1,2], YY[1,3], YY[1,4] and YY[2,1]/H, YY[2,2]/H, YY[2,3]/H and YY[2,4]/H, respectively, in the integration procedure DIFSUB.

When we proceed to integrate the system of equations, Eqs. (7)-(10), from the sonic radius inward towards the pellet surface, we expect a steep drop in the values of q and E. On the other hand, when we integrate from the sonic radius outward, we expect a slow rise in the values of q and E towards the ambient plasma condition of q_o and E_o. Based on these considerations, we adopted the DIFSUB method[3] to carry out the integration. This is because this integration procedure is essentially a predicator-corrector method and the integration steps are self-adjustable.

By using the procedure RISOE/DIFSUB/A[4], the system of equations, Eqs. (7)-(10), was integrated numerically. The boundary condition, Eq. (31), at the far downstream region, however, was replaced by

$$M = \sqrt{5/\gamma} \qquad \text{for } r \geq r \text{ .}$$

A program named PELREF (abbreviation for pellet refuelling) was then written in the Algol language. The general computational results for a range of E_* (1×10^2 - 4×10^4 eV) are shown in Table II. In the table, $\hat{r}$ is normalized pellet radius, i.e. $\hat{r} = r_p/r_*$ and r_M is the normalized ablated cloud boundary where the flow Mach number is within 10^{-3} of its asymptotic value.

To display the results obtained, three different types of plotting programs were also written.

The PELREF/CURVE shows the computed results in graphic form from the pellet surface outward to a radial distance of seven times the sonic radius. Some typical results are shown in Fig. 1a and Fig. 2a. A detailed description of the program is given in the appendix.

PELREF/CL shows the results of the subsonic region in further detail.

PELREF/ASYMP shows the results in the supersonic region from the sonic radius $r = 1$ to a distance of $r = 25$ and compares the results with the asymptotic solutions of Section 2.5. Computational results indicate that for an ablatant with specific heat ratio $\gamma = 1.4$ in the range of E_* investigated, the flow parameters already reach their asymptotic values at a distance of ten times the sonic radius. Typical results are shown in Fig. 1b and Fig. 2b, respectively.

<u>Table I</u>. Corresponding notations used in the analysis and in the program.

Symbols used in the analysis	Symbols used in the program	
$L(E)$	L	
$\sigma(E)$	S	
$\lambda(E)$	A	
E_*	ES	
$\Lambda_* \equiv \Lambda(E_*)$	AS	
$\Lambda'(E) \equiv \Lambda(E)/\Lambda_*$	AO	
$L(E_*)$	LS	
$\sigma(E_*)$	SS	
$(d\sigma/dE)E = E_*$	DS	
λ_*	C	
N_*	NS	
γ	G	
r	X	
v	Y1,	YY[1,1]
θ	Y2,	YY[1,2]
q	Y3,	YY[1,3]
E'	Y4,	YY[1,4]
dv/dr	Z[1],	YY[2,1]/H
$d\theta/dr$	Z[2],	YY[2,2]/H
dq/dr	Z[3],	YY[2,3]/H
dE'/dr	Z[4],	YY[2,4]/H

Table II. Summary of computational results ($\gamma = 1.4$)

Es (eV)	As	Ns	C	$\tilde{q}$	$\tilde{E}$	f	r_M	kT_o (eV)
1.0 E 02	1.8080 E-16	-0.37499	0.9203	1.548	1.181	0.5990	93	5.9050 E 01
3.0 E 02	4.3255 E-17	-0.60708	0.9712	1.557	1.212	0.6509	48	1.8180 E 02
5.0 E 02	2.0633 E-17	-0.66295	0.9853	1.557	1.218	0.6623	43	3.0450 E 02
7.5 E 02	1.1133 E-17	-0.69458	0.9904	1.560	1.218	0.6697	38	4.5675 E 02
1.0 E 03	7.0809 E-18	-0.70825	1.0016	1.560	1.218	0.6717	38	6.0900 E 02
2.0 E 03	2.2810 E-18	-0.70755	0.9961	1.557	1.204	0.6750	38	1.2040 E 03
5.0 E 03	4.7948 E-19	-0.64405	0.9844	1.561	1.178	0.6659	51	2.9450 E 03
7.0 E 03	2.6786 E-19	-0.60967	0.9761	1.561	1.167	0.6604	61	4.0845 E 03
3.0 E 04	2.1256 E-20	-0.44642	0.9361	1.560	1.119	0.6285	107	1.6785 E 04
4.0 E 04	1.2882 E-20	-0.41598	0.9286	1.559	1.111	0.6218	128	2.2220 E 04

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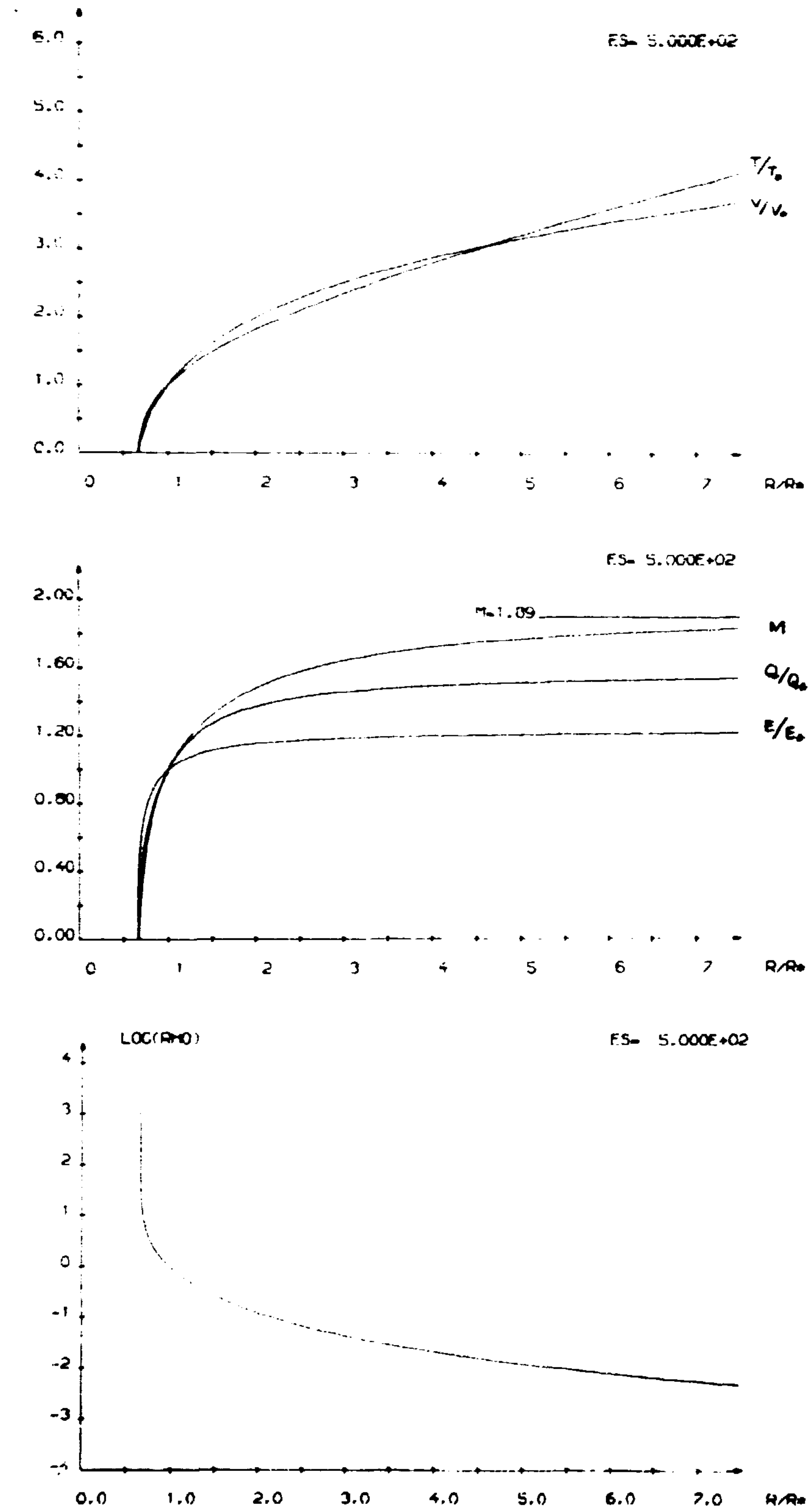

Fig. 1a. Dimensionless temperature, T/T_*, flow velocity, v/v_*, density, RHO = ρ/ρ_*, Mach number, M, incident electron energy flux, Q/Q_* and incident electron energy E/E_* versus dimensionless radius R/R_*. Incident electron energy at the sonic radius R_*, ES = 500 eV (kT_0 = 305 eV). The surface of the pellet is located at $\hat{r}$ = 0.6623.

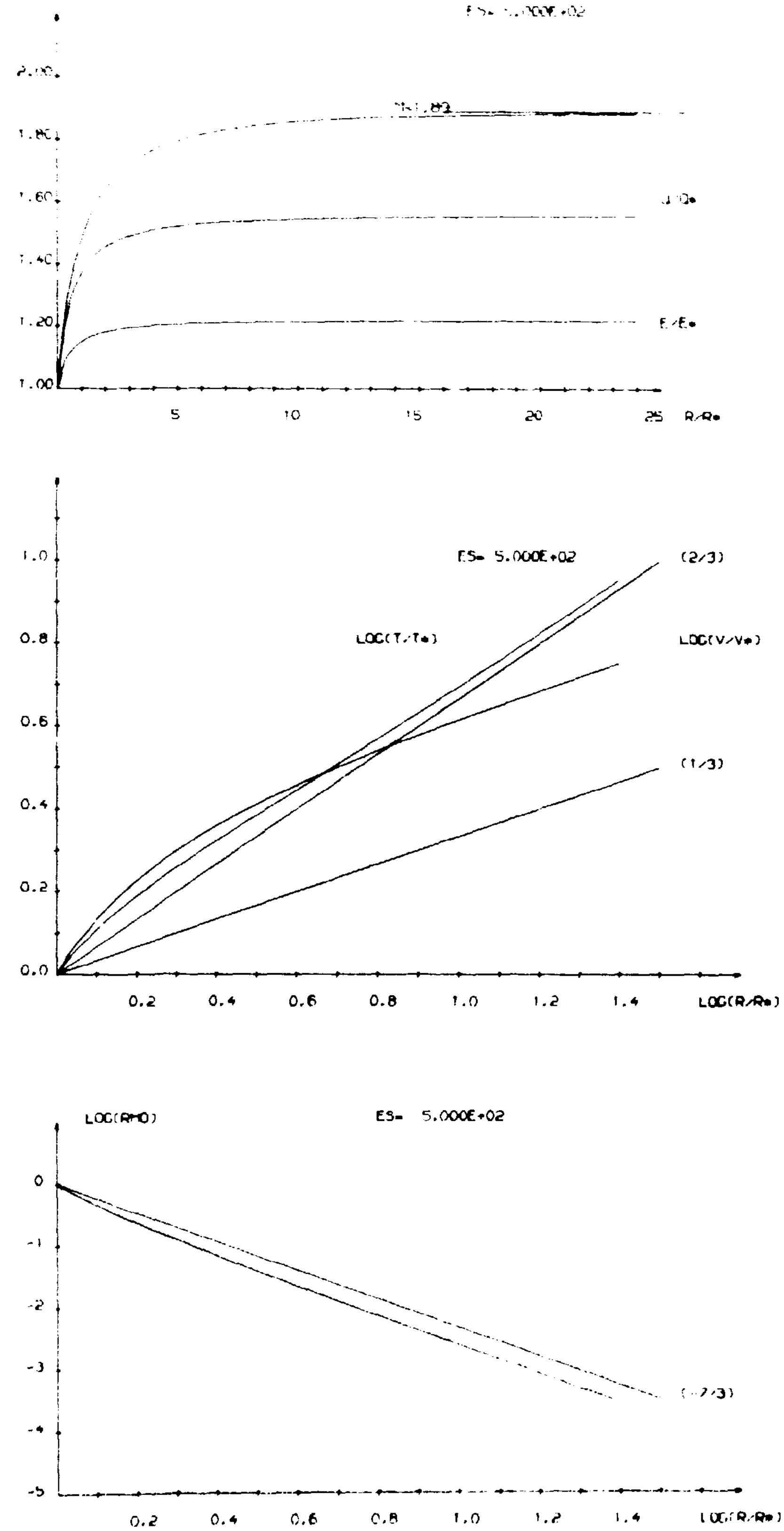

Fig. 1b. Comparison of the dimensionless flow parameters v/v_*, T/T_* and ρ/ρ_* in the supersonic region obtained from the numerical integration with their corresponding asymptotic solutions (curves denoted by the bracket). Incident electron energy at the sonic radius ES = 500 eV. The asymptotic values of Q/Q_* and E/E_* are 1.557 and 1.218, respectively.

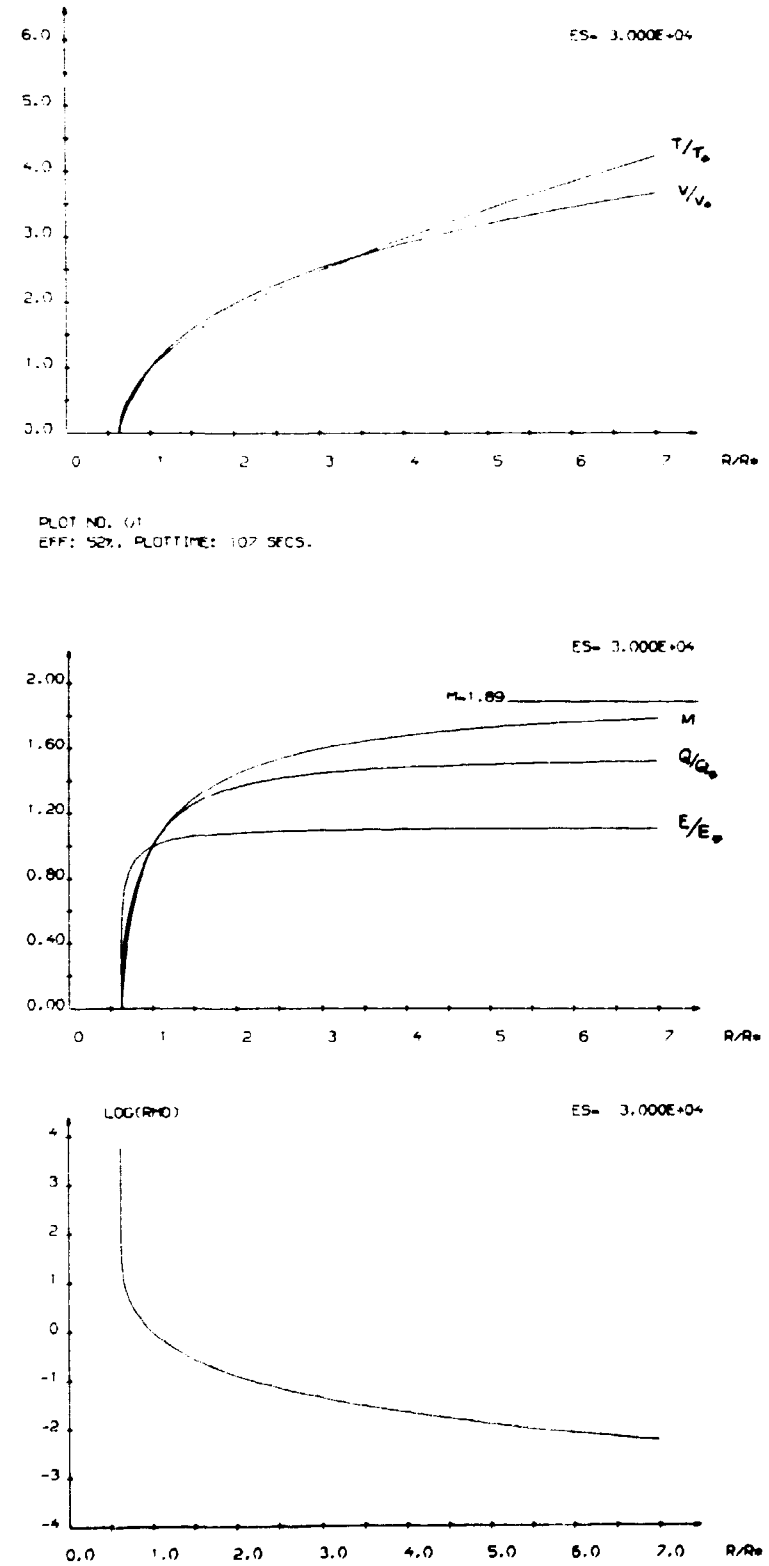

Fig. 2a. Dimensionless flow parameters of the ablatant, incident electron energy and energy flux versus dimensionless radius, R/R_*. Incident electron energy at the sonic radius, $ES = 3\times10^4$ eV ($kT_o = 1.68\times10^4$ eV). The surface of the pellet is located at $\hat{r} = 0.6285$.

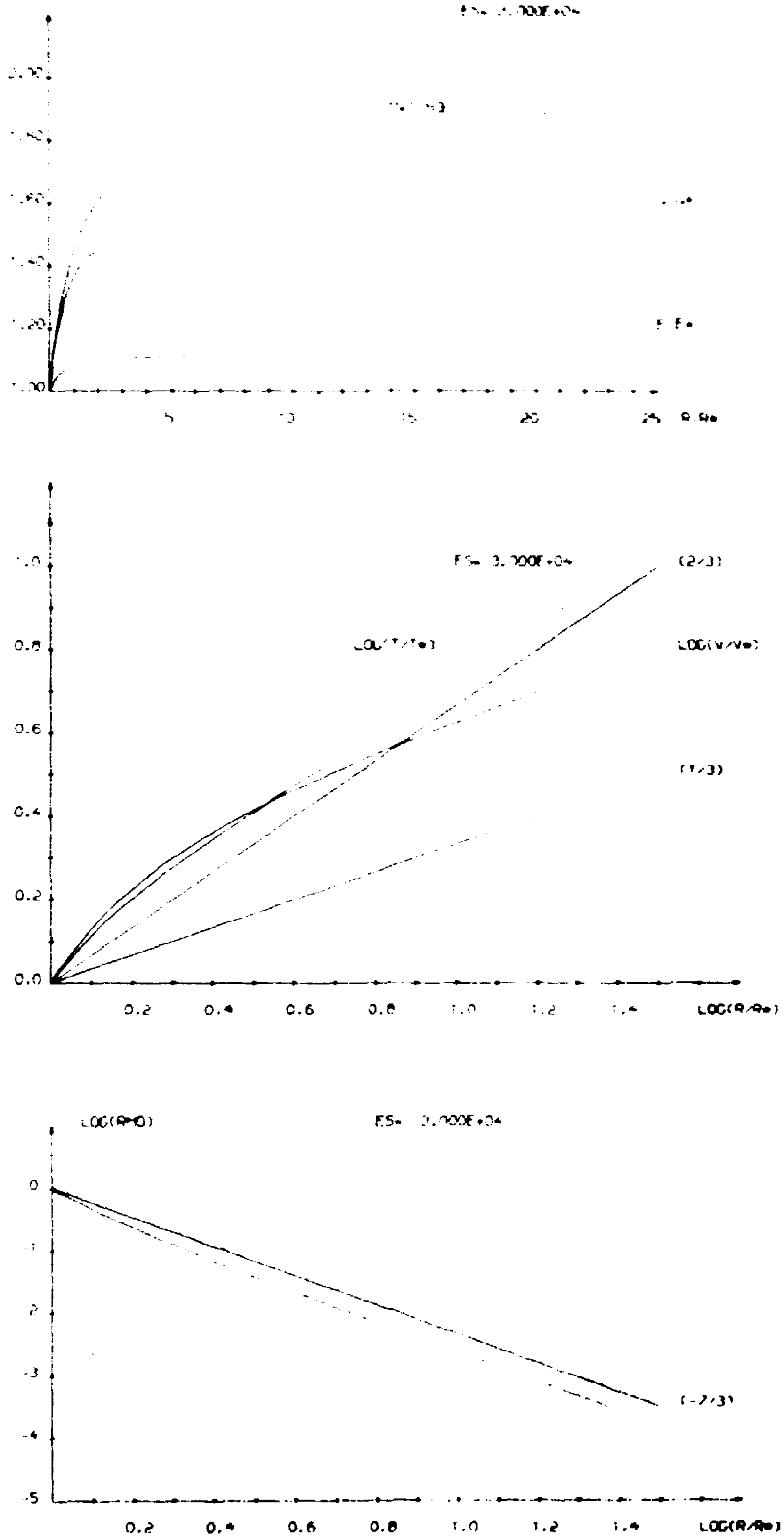

Fig. 2b. Comparison of the dimensionless flow parameters V/V_*, T/T_* and ρ/ρ_* in the supersonic region obtained from the numerical integration with their corresponding asymptotic solutions (curves denoted by the brackets). Incident electron energy at the sonic radius ES = 3×10^4 eV. The asymptotic values of Q/Q_* and E/E_* are 1.560 and 1.119, respectively.

APPENDIX

Detailed description of the program PELREF/CURVE and the accompanying procedure PELAB.

PELREF/CURVE (05/06/81)

```
$ SET INSTALLATION
BEGIN
 COMMENT :::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::
  :        THIS PROGRAM USES THE PROCEDURE PELAB TO PLOT THE            :
  :        COMPUTATED RESULTS FROM THE PELLET SURFACE UP TO A           :
  :        DISTANCE OF SEVEN TIMES THE SONIC RADIUS                     :
  ::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::: ;

$ INCLUDE "RISOE/PLOTSTART/A"
$ INCLUDE "RISOE/PLOTAXES/A"
$ INCLUDE "RISOE/PLOTLINE/A"
  FILE  INP (KIND =DISK,FILETYPE=7), OUT (KIND =PRINTER);
  REAL  ES,AS,C,K,H,XMAX,XMIN,RHOMIN,RHOMAX,QMAX,EMAX,PR,NP;
  INTEGER  I,IMAX,H,BMAX,N,NL,J;
  ARRAY  XS[0:200],XL[0:200],RS[0:9,0:200],RL[0:9,0:200];
$ INCLUDE "PELAB/A"

  WRITE(OUT,</"PROGRAM PELREF/CURVE"//>);

COMMENT :::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::
 :        THE INPUT DATA IS GIVEN BY DATAPEL                             :
 :::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::: ;
  READ(INP,/,ES,K,XMAX,PR,NP);
  WRITE(OUT,<"ES=",E10.3, "  K=",F8.5, "  XMAX=",F6.2>,ES,K,XMAX);
  PELAB(ES,K,XMAX,PR,NP);

COMMENT ::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::
 :        NOW BEGIN THE PLOTTING                                      :
 :::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::;

   DELTAX:= 0.75/200;  DELTAY:=0.1/50;
   SETORIGO(0.3,0);
  PLOTAXES(0,0,0,7.5,0,2,2,0.5,0.2);
  PLOTSTRING((STRINGBUFF[*],<"R/R*">),7.8,-0.2,0.3);
  FOR M:=7 STEP -1 UNTIL 0 DO
  PLOTSTRING((STRINGBUFF[*],<I3>,M),M-0.15,-0.2,0.3);
  FOR M:=0 STEP 0.4 UNTIL 2.0 DO
  PLOTSTRING((STRINGBUFF[*],<F5.2>,M),-0.6,M,0.3);
  PLOTSTRING((STRINGBUFF[*],<"ES=",E10.3>,ES),6,2.2,0.3);
  PLOTSTRING((STRINGBUFF[*],<"M=1.89">),4.5,1.89,0.3);
  PLOTLINE (5.2,1.89,7.5,1.89);
   XS[0]:=1;  FOR I:=1,2,3,4,5,6  DO RS[I,0]:=1;
   XL[0]:=1;  FOR I:=1,2,3,4,5,6  DO RL[I,0]:=1;
 FOR J:=3,4,6 DO
  BEGIN  CASE J OF
   BEGIN  3:SETCHAR("3"); 4:SETCHAR("4"); 6:SETCHAR("6"); END;
   FOR I:=IMAX STEP -1 UNTIL 1 DO
   PLOTLINE(XS[I],RS[J,I],XS[I-1],RS[J,I-1]);
   FOR B:=0 STEP 1 UNTIL BMAX-1 DO
   PLOTLINE(XL[B],RL[J,B],XL[B+1],RL[J,B+1]);
  END;
   DELTAX:=0.1;  DELTAY:=0.1;
$ :::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::
STARTPLOT;
   DELTAX:= 0.75/200;  DELTAY:=0.1/20;
   SETORIGO(0.3,0);
  PLOTAXES(0,0,0,7.5,0,6.5,0.5,0.5);


  PLOTSTRING((STRINGBUFF[*],<"R/R*">),7.8,-0.5,0.3);
  FOR M:=7 STEP -1 UNTIL 0 DO
  PLOTSTRING((STRINGBUFF[*],<I3>,M),M-0.15,-0.5,0.3);
  FOR M:=0 STEP 1.0 UNTIL 6 DO
  PLOTSTRING((STRINGBUFF[*],<F4.1>,M),-0.6,M,0.3);
  PLOTSTRING((STRINGBUFF[*],<"ES=",E10.3>,ES),6,6,0.3);
   XS[0]:=1;  FOR I:=1,2,3,4,5,6  DO RS[I,0]:=1;
   XL[0]:=1;  FOR I:=1,2,3,4,5,6  DO RL[I,0]:=1;
  FOR J:=1,2 DO
   BEGIN  CASE J OF
    BEGIN  1:SETCHAR("1"); 2:SETCHAR("2"); END;
    FOR I:=IMAX STEP -1 UNTIL 1 DO
    PLOTLINE(XS[I],RS[J,I],XS[I-1],RS[J,I-1]);
    FOR B:=0 STEP 1 UNTIL BMAX-1 DO
    PLOTLINE(XL[B],RL[J,B],XL[B+1],RL[J,B+1]);
   END;
    DELTAX:=0.1;  DELTAY:=0.1;
$ :::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::
STARTPLOT;
   N:= ENTIER(LOG(ABS(RHOMIN)))-1;  WRITE(OUT,<"N=",I3>,N);
   NL:=ENTIER(LOG(RHOMAX))+1;  WRITE(OUT,<"NL=",I3>,NL);
   DELTAX:=0.75/200; DELTAY:=0.1*(NL-N)/120;
   SETORIGO(0.3,-N);
  PLOTAXES(0,N,0,7.5,N,NL+0.4,0.5,1.0);
  PLOTSTRING((STRINGBUFF[*],<"R/R*">),7.8,N-0.65,0.3);
  FOR M:=7 STEP -1 UNTIL 0 DO
  PLOTSTRING((STRINGBUFF[*],<F4.1>,M),M-0.15,N-0.65,0.3);
  FOR M:=N STEP 1 UNTIL NL DO
  PLOTSTRING((STRINGBUFF[*],<I3>,M),-0.45,M,0.3);
  PLOTSTRING((STRINGBUFF[*],<"LOG(RHO)">),0,45,NL+0.4,0.3);
  PLOTSTRING((STRINGBUFF[*],<"ES= ",E10.3>,ES),6,NL+0.4,0.3);
   SETCHAR("*");
   XS[0]:=1; RS[5,0]:=1;
  FOR I:=IMAX STEP -1 UNTIL 1 DO
  PLOTLINE(XS[I],LOG(ABS(RS[5,I])),XS[I-1],LOG(ABS(RS[5,I-1])));
   XL[0]:=1; RL[5,0]:=1;
  FOR B:=0 STEP 1 UNTIL BMAX-1 DO
  PLOTLINE(XL[B],LOG(RL[5,B]),XL[B+1],LOG(RL[5,B+1]));

$ INCLUDE "RISOE/PLOTSLUT/A"
END OF PROGRAM.
```

WORKFILE: PELAB/A (??/??/??)

```
PROCEDURE  PELAB(ES,K,XMAX,PR,NP);
            VALUE ES,K,XMAX,PR,NP;  REAL ES,K,XMAX,PR,NP;

BEGIN  COMMENT ::::::::::::::::::::::::::::::::::::::::::::::::::::::::::
        :        THIS PROCEDURE USES THE DIFSUB METHOD TO CALCULATE :
        :        THE ABLATION OF A REFUELLING PELLET                :
        ::::  ::::::::::::::::::::::::::::::::::::::::::::::::::::::::

LABEL  SLUT,REP,LEFT;
INTEGER  KFLAG,JSTART;
REAL X,ZS,LS,IS,DS,SS,RHO,MACH,P;
REAL H,HS,HMIN,HMAX,EPS,MF;
ARRAY SAVE,YY[0:8,0:4],PW[0:4,0:4],YMAX,ERROR,Y,Z,INT[0:4];
PROCEDURE DERIV(X,Y,PW,N);VALUE N;INTEGER N;ARRAY X,Y[*],PW[*,*];
BEGIN END;
PROCEDURE F(X,Y,Z);  VALUE X; REAL X;  ARRAY Y,Z[0];
  BEGIN
REAL E,L,S,A,AO,Y1,Y2,Y3,Y4;
 Y1:=Y[1]; Y2:=Y[2]; Y3:=Y[3]; Y4:=Y[4];
  IF Y1>0 AND X>0 THEN
  BEGIN
    E:=ES*Y4;
    L:= 1/ (2.35@14 + 4@11*E + 2@17/E**2);
    S:= IF E>100 THEN 8.8@-13/E**1.71 + 1.62@-12/E**1.932
       ELSE  1.13@-14/E;
    A:= L*L/L + S;
    AO:= A/AS;
  IF Y2-Y1**2=0  THEN Z[1]:=ZS ELSE
       Z[1]:=2*Y1*Y2/(Y2-Y1**2)*(AO*Y3/(Y2*Y1)-1/X);
       Z[2]:=2*AO*Y3/Y1 - 0.4*Y1*Z[1];
       Z[3]:=C*AO*Y3/(Y1*X**2);
       Z[4]:=2*C*L/(ES*AS)/(Y1*X**2);
  END
  ELSE GO TO SLUT;
 END;
$ INCLUDE "RISOE/DIFSUB/A"

ZS:= 2/3*(1+SQRT(2.875 - 1.875*K));
 WRITE(OUT,<"ZS=",E11.4>,ZS);
LS:= 1/(2.35@14+4@11*ES+2@17/ES**2); WRITE(OUT,<"LS=",F10.3>,LS);
SS:= IF ES>100  THEN 8.8@-13/ES**1.71+1.62@-12/ES**1.932
     ELSE 1.13@-14/ES;
 WRITE(OUT,<"SS=",E10.3>,SS);
AS:= 2*LS/ES + SS;
 WRITE(OUT,<"AS=",E11.4>,AS);
DS:= IF ES>10  THEN -1.504@-12/ES**2.71+3.1298@-12/ES**2.932
     ELSE -1.13@-14/ES**2;
 WRITE(OUT,<"DS=",E10.3>,DS);
NS:=-2*LS/AS**2*(2*LS/ES**2*((4@11*ES-4@17/ES**2)*LS + 1)-DS);
 WRITE (OUT,<"NS=",E11.4>,NS);
C:=K/( 1-NS );
 WRITE(OUT [SPACE  3 ],<"C =",E11.4>,C);

COMMENT  ::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::
  :        NOW BEGIN THE INTEGRATION                                :
  :::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::


  EPS:=1@-4; INT[0]:=1; HMAX:=5; H:=1@-4; HS:=H;
REP:
  JSTART:=0; MF:=2;
  YMAX[1]:=YMAX[2]:=YMAX[3]:=YMAX[4]:=1;
  X:=1; YY[1,1]:=YY[1,2]:=YY[1,3]:=YY[1,4]:=1;
  FOR I:=1,2,3,4 DO SAVE[1,I]:= IF H<0 THEN 2 ELSE 0.5;
 WRITE(OUT [SPACE 2 ],< X8,"X",X12,"V",X12,"T",X12,"Q",X12,"E*",X10,
      "D(V)",X9,"D(T)",X9,"D(Q)",X9,"D(E*)",X9,"H">);
  FOR I:=1 STEP 1 UNTIL 4 DO Y[I]:=YY[1,I];
  F(X,Y,Z);
 WRITE(OUT,<10(E13.4)>,X,Y[1],Y[2],Y[3],Y[4],Z[1],Z[2],Z[3],Z[4],H);
 $ :::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::
  IF H<0 THEN GO TO LEFT ELSE
    BEGIN
  FOR I:=1,I+1 WHILE X<XMAX AND MACH<SQRT(25/7) AND
      SIGN(H)=SIGN(HS) DO
BEGIN
   RHOMIN:=1;  QMAX:=1;  EMAX:=1;
  DIFSUB(4,X,YY,SAVE,H,HMIN,HMAX,EPS,MF,YMAX,ERROR,KFLAG,JSTART,
          INT,PW,F,DERIV);
  RHO:= 1/(YY[1,1]*X**2);
  P:=RHO*YY[1,2];
  MACH:= YY[1,1]/SQRT(YY[1,2]);
   X,[I]:=X;
   RS[1,I]:=YY[1,1];  RS[2,I]:=YY[1,2];  RS[3,I]:=YY[1,3];
   RS[4,I]:=YY[1,4];  RS[5,I]:=RHO;  RS[6,I]:=MACH;
   RHOMIN:=MIN(RHOMIN,RHO);   QMAX:=MAX(QMAX,YY[1,3]);
   EMAX:=MAX(EMAX,YY[1,4]);  IMAX:=I;
END;
 WRITE(OUT [SPACE 2 ],< X8,"X",X12,"V",X12,"T",X12,"Q",X12,"E*",X10,
      "RHO",X9,"MACH",X9,"D(Q)",X9,"D(E*)",X9,"H">);
 WRITE(OUT,<10(F13.4)>,X,YY[1,1],YY[1,2],YY[1,3],YY[1,4],
          RHO,MACH,YY[2,3]/H,YY[2,4]/H,H);
 WRITE(OUT [SPACE 2 ],<" ">);
 WRITE(OUT[SPACE 2 ],<"RHOMIN=",E13.4>,RHOMIN);
 WRITE(OUT[SPACE 2 ],<"QMAX=",E13.4>,QMAX);
 WRITE(OUT[SPACE 2 ],<"EMAX=",E13.4>,EMAX);
 WRITE(OUT[SPACE 2 ],<"IMAX=",F6.2>,IMAX);
    END;
   H:=-HS;  HMAX:=0.1;  GO TO REP;
 $ :::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::
LEFT:
  FOR B:=1,B+1 WHILE YY[1,1]<SAVE[1,1] AND YY[1,3]>0  DO
BEGIN
   RHOMAX:=0;  XMIN:=1;
  DIFSUB(4,X,YY,SAVE,H,HMIN,HMAX,EPS,MF,YMAX,ERROR,KFLAG,JSTART,
          INT,PW,F,DERIV);
  RHO:= 1/(YY[1,1]*X**2);
  P:=RHO*YY[1,2];
  MACH:= YY[1,1]/SQRT(YY[1,2]);
   XL[B]:=X;
   RL[1,B]:=YY[1,1]; RL[2,B]:=YY[1,2]; RL[3,B]:=YY[1,3];
   RL[4,B]:=YY[1,4]; RL[5,B]:=RHO;  RL[6,B]:=MACH;
   RHOMAX:=MAX(RHOMAX,RHO); XMIN:=MIN(XMIN,X); BMAX:=B;
END;
 WRITE(OUT [SPACE 2 ],< X8,"X",X12,"V",X12,"T",X12,"Q",X12,"E*",X10,
      "RHO",X9,"MACH",X9,"D(Q)",X9,"D(E*)",X9,"H">);
 WRITE(OUT,<10(F13.4)>,X,YY[1,1],YY[1,2],YY[1,3],YY[1,4],
          RHO,MACH,YY[2,3]/H,YY[2,4]/H,H);
 WRITE(OUT [SPACE 2 ],<" ">);


 WRITE(OUT[SPACE 2 ],<"RHOMAX=",F6.  MAX);
 WRITE(OUT[SPACE 2 ],<"XMIN=",E11.4>,XMIN);

SLUT:
 WRITE(OUT [SPACE 2 ],< X8,"X",X12,"V",X12,"T",X12,"Q",X12,"E*",X10,
      "RHO",X9,"MACH",X9,"D(Q)",X9,"D(E*)",X9,"H">);
 WRITE(OUT,<10(F13.4)>,X,YY[1,1],YY[1,2],YY[1,3],YY[1,4],
          RHO,MACH,YY[2,3]/H,YY[2,4]/H,H);
END PELAB;
```

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Title and author(s)

PELREF

(A numerical code for computing the ablated state of a refuelling pellet)

C.T. Chang

Date May 1981

Department or group

Physics

Group's own registration number(s)

27 pages + tables + illustrations

Abstract

Assuming a constant specific heat ratio and using the neutral shielding model, this report presents a numerical code for calculating the ablation rate and the state of the ablatant of a refuelling pellet. Results are given for plasma conditions corresponding to present and to future toridal devices.

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